COMPARING THE RELATIVE LIKELIHOOD OF EVENTS: THE FALLACY OF COMPOSITION

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The objective of this article is to contribute to the documented dearth of research on teachers' probabilistic knowledge. Prospective teachers of elementary school mathematics were asked to identify which result from five flips of a fair coin was least likely to occur. Participants, instead of being presented with sequences, were presented with events, that is, sets of outcomes, for five flips of a fair coin. A particular logical fallacy, the fallacy of composition, was used to analyze the juxtaposition of responses and response justifications. As a result, the fallacy of composition was found in the response justifications for both normatively incorrect and correct relative likelihood comparisons. Combing the results of this and prior research utilizing the fallacy of composition demonstrates that logical fallacies are a burgeoning area of research for those investigating relative likelihood comparisons and teachers' probabilistic knowledge.

Keywords: Probability; Teacher Knowledge

Given there is a limited amount of research on "teachers' probabilistic knowledge" (Jones, Langrall & Mooney, 2007, p. 933), one objective of this article is to continue an emergent thread of research investigating prospective mathematics teachers' probabilistic knowledge (Chernoff, in press, 2012a-c, 2011; Chernoff & Russell, 2012a, 2012b, 2011a, 2011b). A second objective of this article is to contribute to a well established domain of research, which accounts for normatively incorrect responses to relative likelihood comparisons (e.g., Abrahamson, 2009; Borovcnik & Bentz, 1991; Chernoff, 2009; Cox & Mouw, 1992; Hirsch & O'Donnell, 2001; Kahneman & Tversky, 1972; Konold, 1989; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; LeCoutre, 1992; Rubel, 2007; Shaughnessy, 1977, 1981; Tversky & Kahneman, 1971, 1974). Yet a third objective of this article is to bolster the results of a particular research thread previously published in the conference proceedings of the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Chernoff & Russell, 2011a) and the International Group for the Psychology of Mathematics Education (Chernoff & Russell, 2011b)

To meet the objectives stated above, prospective teachers of mathematics, as was the case in prior research, were asked to make a comparison of relative likelihood; however, unlike previous research (for exceptions see, for example, Chernoff, in press; Chernoff & Russell, 2011b), participants were asked not to compare the relative likelihood of sequences, but, rather, events (i.e., sets of outcomes) resulting from five flips of a fair coin. Also breaking from tradition, this research, instead of using the theories, models and frameworks most frequently associated with relative likelihood comparison research (e.g., Kahneman & Tversky, 1972; Konold et al., 1993), uses a particular logical fallacy for the analysis of results. The current set of responses to a task recently introduced to the research literature – the relative likelihood of events task (Chernoff & Russell, 2011b) – once again evidence use of the fallacy of composition (i.e., because parts of a whole have a certain property, it is argued that the whole has that property). Also once again, the fallacy of composition is evident in both incorrect and correct response justifications in the current research. The results from this current research, coupled with the results from the initial study (Chernoff & Russell, 2011b), further validate that certain prospective teachers of mathematics use the fallacy of composition when making relative likelihood comparisons. In line with this point, it will, further, be argued that logical fallacies are a

viable venue for future research investigating relative likelihood comparisons and teachers' probabilistic knowledge.

A Brief Summary Of Prior Research

Research into probabilistic thinking and the teaching and learning of probability has, in the past, seen a focus on normatively incorrect responses. Specific to the field of mathematics education, the theories, models and frameworks associated with heuristic and informal reasoning – rooted in the notions of conceptual analysis (Thompson, 2008; Von Glaserveld, 1995), grounded theory (Strauss & Corbin, 1998) and abduction (Lipton, 1991; Peirce, 1931) – have, traditionally, accounted for normatively incorrect responses to probabilistic tasks. Chernoff (in press, 2012a-c, 2011) and Chernoff and Russell (2012a, 2012b, 2011a, 2011b) have recently provided detailed accounts of (the history of) the theories, models and frameworks associated with heuristic and informal reasoning in the field of mathematics education and are (given the boundaries associated with the present venue) recommended to the reader.

More recently, a burgeoning area of research suggests that fallacious reasoning, more specifically, the use of logical fallacies, can account for certain normatively incorrect responses to relative likelihood comparisons. For example, Chernoff (in press) and Chernoff and Russell (2011b) utilized the fallacy of composition (when an individual infers something to be true about the whole based upon truths associated with parts of the whole) as a framework to anlayse relative likelihood comparison responses. Prospective mathematics teachers were asked to determine which of five possible coin flip sequences were least likely to occur. As was the case in similar research (e.g., Chernoff & Russell, 2011a), the fallacy of composition accounted for normatively incorrect responses to the task. More specifically, the researchers demonstrated that participants referenced the equiprobability of the coin, noted that the sequence is comprised of flips of a fair coin and, as such, fallaciously determined that the sequence of coin flips should also have a heads to tails ratio of one to one. In other words, the properties associated with the fair coin (the parts), which make up the sequence (the whole), are (fallaciously) expected in the sequence. As a result, the fallacy of composition, in addition to the traditional theories, models and frameworks associated with heuristic and informal reasoning, accounted for certain normatively incorrect responses to a probabilistic task.

Chernoff and Russell (2012a, 2011b) further established that certain prospective mathematics teachers, when asked to identify which event (i.e., outcome or subset of the sample space) from five flips of a fair coin was least likely to occur, did not use the representativeness heuristic (Kahneman & Tversky, 1972), the outcome approach (Konold, 1989) or the equiprobability bias (Lecoutre, 1992). Instead, they again demonstrated that participants utilized the fallacy of composition. Worthy of note, the fallacy of composition accounted for both normatively correct and incorrect responses to the new relative likelihood comparison task. As mentioned, this article extends previous research utilizing the fallacy of composition.

The Fallacy Of Composition

The fallacy of composition, as a theoretical framework for the analysis of response justifications for relative likelihood comparisons, has revealed a new approach to account for normatively incorrect responses. Moreover, the fallacy of composition has proven a viable approach for both relative likelihood comparisons between sequences and events. Given that the current research tasks participants with relative likelihood comparisons between events, the fallacy will be used for the impending analysis of results.

Essentially, the *fallacy of composition* occurs when an individual infers something to be true about the *whole* based upon truths associated with *parts of the whole*. For example: Bricks (i.e., the parts) are sturdy (i.e, the truth). Buildings (i.e., the whole) are made of bricks (i.e., the parts). Therefore, buildings (i.e., the whole) are sturdy (which is not necessarily true).

As will be demonstrated throughout the analysis of results, certain participants in this research inferred certain truths associated with individual coin flips to be true for events, that is, sets of outcomes. Prior to this analysis, however, certain methodological components (e.g., Task and Particpants) are discussed.

Task

As seen in Figure 1 below, the task used in this research is the same task recently introduced to the research literature (Chernoff & Russell, 2011b).

Which of the following is the least likely result of five flips of a fair coin?
a) three heads and two tails
b) four heads and one tail
c) both results are equally likely to occur
Justify your response...

Figure 1: The Relative Likelihood Of Events Task

Unlike research involving sequences of coin flips, the relative likelihood of events task possesses a normatively correct response (i.e., "four heads and one tail") and two normatively incorrect responses (i.e., "three heads and two tails" and "both results are equally likely to occur"). See, for example, Chernoff and Russell (2011b) for further details associated with history and design of both relative likelihood comparison tasks and the relative likelihood of events task.

Participants

Participants in our research were (n =) 54 prospective mathematics teachers enrolled in a methods course designed for teaching elementary school mathematics. More specifically, the 54 participants were comprised of two classes, containing 26 and 28 students, taught by the same instructor. Participants were presented with the relative likelihood of events task and were allowed to work on the task until completion. Of note, the participants had not answered any of the other versions of the relative likelihood task prior. Further, the topic of probability had yet to be discussed in class at the time of the research.

Results

Participants' responses fell into three categories. The majority, 38 of the 54 participants (or 70%), incorrectly declared that the events (three heads and two tails and four heads and one tail) were equally likely to occur. Further, fourteen of the 54 participants (or 26%) correctly declared that four heads and one tail was least likely (of the events presented in the task) to occur and two participants (or 4%) declared that three heads and two tails was least likely to occur.

Table 1: The Numerical Breakdown Of Responses

Number of participants	Three heads and two tails	Four heads and one tail	Equally likely
Class A (26)	1	8	17
Class B (28)	1	6	21
Total (54)	2	14	38
Percentage (100)	**4%**	**26%**	**70%**

Given the boundaries associated with the present venue (i.e., the 8 page limitation), the responses from eleven individuals – five from individuals that declared the events equally likely to occur and

six from individuals that declared four heads and one tails as least likely – are now featured in the analysis of results.

Analysis Of Results

The response justifications from certain individuals reveal the fallacy of composition when comparing the relative likelihood of the events they are presented. Given that use of the fallacy of composition is witnessed in the justifications for both incorrect and correct responses to the task and, as such, the analysis of results is organized accordingly.

Both Results Are Equally Likely To Occur (Incorrect Response)

Responses from (more than) 5 of the 38 individuals that declared the events were equally likely to occur evidence (to varying degrees) use of the fallacy of composition. Consider, for example, the responses from Marie Hank and Walter, who, respectively, declare that "either outcome could occur equally," "both outcomes are equally likely" and "it is just as likely for either outcome" because of (for all three of them) the equiprobability of flipping a fair coin.

> *Hank:* Both outcomes are equally likely to occur because the probabilty of getting a head or tail is the same.
>
> *Marie:* either outcome could occur equally. each time the coin is flipped there is a 50 50 chance that the coin will land with a certain face up.
>
> *Walter:* It is just as likely for either outcome or any possible outcome (1H+4T, 2H+3T, 3H+2T, 4H+1T) since there is an equal chance of either occuring on each flip.

Discussed in terms of the theoretical framework, although Marie and Frank declare that the "outcome(s)" (i.e., the building) are equally likely (i.e., sturdiness of building) because of the equiprobability of the coin (i.e., sturdiness of the brick), and although Marie makes reference to the equiprobability each time the coin is flipped, their responses do not (explicitly) declare that the "outcome(s)" they are discussing are comprised of five flips of the coin (i.e., the building is made of bricks). Walter extends the argument of Hank and Marie to further declare that not only are the two outcomes they are presented are equally likely, but, further, "any" (of course, Walter is "missing" the all heads and all tails outcomes) of the outcomes are equally likely because of the equiprobability associated with the flip of a fair coin.

The responses of Skyler, Junior and Walter (differently) reference that the events are comprised of five flips of a fair coin. Consider, then, the responses from Sklyer and Junior.

> *Skyler:* Each flip of the coin has the same odds of having heads land face up as having tails land face up. If you flip the coin once, there is a 50% chance you'll get a tail, and 50% chance you get a heads. The next time you flip the coin, the result is not dependent or related to the result of the first flip in any way, so the odds of flipping a tails the second time is still 50%, regardless of what the outcome was the first flip (same goes for head). This continues for all five flips.
>
> *Junior:* on each flip of the coin there exists a 50:50 chance of heads or tails occurring. therefore, the combinations even when aggregated together have the same chance of occurring.

Like Hank and Marie, Skyler and Junior, in their responses, contend that the events are equiprobable because of the equiprobability associated with the coin flip(s). Discussed in terms of the fallacy of composition, the responses from Sklyer and Junior reference the equiprobability (i.e., sturdiness) associated with each coin flip (i.e., brick); further, their responses reference that the event (i.e., the building) is comprised of coin flips (i.e., bricks); and, as such, the outcome(s) should be equiprobable (i.e., the building made of sturdy bricks should be sturdy) or, in the words of Junior, "even when aggregated together have the same chance of occurring." Skyler and Junior (and, arguably, Hank and Marie) inferred the equiprobability of the events based upon the equiprobability associated with the flip of a fair coin. Put in more general terms, they used the fallacy of composition when determining which event was least likely to result from five flips of a fair coin.

Four Heads And One Tail (Correct Response)

As presented above, the responses from certain individuals that incorrectly declared both results equally likely to occur demonstrate use of the fallacy of composition when comparing the relative likelihood of events. As will be presented below, the responses from certain individuals that correctly determined that four heads and one tail was least likely (of the events presented) to occur also evidence use of the fallacy of composition. However, as will be evidenced in the six responses that follow, the result established through use of the fallacy of composition, that is, that the events are equally likely to occur, is subsequently used to determine that four heads and one tail is least likely to occur. Stated in more general terms, the participants are fallaciously arriving at the "correct" response.

Consider, for example, the similar yet different responses from Gale, Mike and Saul who all make reference to an expected ratio of heads to tails for the outcome.

> *Gale:* Four heads and one tail is least likely because each flip gives you a 50% chance of getting either heads or tails so it is more likely you would get closer to answer a.
>
> *Mike:* a – more likely since 3:2 is closer to 1:1 than 4:1 is. When flipping a coin the outcome should be similar to 1 head to 1 tail although it usually isn't.
>
> *Saul:* Three heads and two tails are more likely b/c there is only 2 options so there is a 50/50 chance of each happening, so it is more likely that the outcomes of heads = tail ratio will be equal or close to equal rather than further apart.

All three of the above responses are similar. For example, all three responses declare that the four heads and one tail outcome is least likely to occur because the ratio of heads to tails, that is, four to one, is farther "away" from the expected ratio of heads to tails, that is, one to one, than the "closer" ratio of heads to tails, that is, three to two. Alternatively stated, as Mike succinctly writes, "[three heads and two tails] – more likely since 3:2 is closer to 1:1 than 4:1 is." Further, also garnered from the responses above, participants (as was the case for participants in the previous section) inferred the equiprobability of the events, that is, the expected one to one ratio of the outcomes, based upon inferred equiprobability associated with the flip of a fair coin.

Discussed in terms of the fallacy of composition, the responses of Gale, Mike and Saul (again to varying degrees) reference: the equiprobability (i.e., the sturdiness) associated with the flip of a fair coin (i.e, the brick); the outcome (i.e., the building) being comprised of five flips of a fair coin (i.e., bricks); and, as such, expected, fallaciously, the outcome (i.e., the building) to have an equal ratio of heads to tails (i.e., the building to be sturdy). Subsequently, Gale, Mike and Saul utilize their fallacious conclusion, that is, both sequences should have a one to one ratio of heads to tails (as is the case for the flip of a fair coin), to analyze which of the outcomes they are presented in "closest" in terms of their expected ratio. In all three instances above, the outcomes with a four to one ratio of heads to tails is declared "further away" and, thus, is "correctly" determined as least likely to occur.

Similar (in conclusion) to the responses of Gale, Mike and Saul, Jesse, Jane and Gus also reference an expected ratio of heads to tails, but, unlike Gale, Mike and Saul, instead of referencing "closeness," reference the unlikeliness or unevenness or inequality associated with obtaining a four to one ratio of heads to tails when expecting a ratio closer to one to one.

> *Jesse:* fair coin would give 50/50 chances so 5 flips should be more equal results than 4:1
>
> *Jane:* You're least likely to flip 4 heads and 1 tail, as opposed to 3 heads and 2 tails. Each flip has a 50% chance of flipping either one. That makes is unlikely that you will flip 4/5 of the same side.
>
> *Gus:* b) because you have a 50/50 chance of getting a head or tail each time you flip. Therefore, getting an even amount of each is more likely than all on one side as in b.

Again, all three of the above responses are similar. The three responses declare that the four heads and one tail outcome is least likely to occur because (with an fallaciously expected ratio of one to one for heads and tails) having "4/5 of the same side" is unlikely or "getting an even amount of each is more likely" or there should "be more equal results than 4:1." Again, the ratio of heads to tail, that is,

one to one, is still farther "away" than three to two, but the terminology used by Jesse, Jane and Gus is different than for Gale, Mike and Saul. Further, also garnered from the responses above, participants (as was the case for participants in the previous section) inferred the equiprobability of the events, that is, the expected one to one ratio of the outcomes, based upon the equiprobability associated with the flip of a fair coin.

Discussed in terms of the fallacy of composition, the responses of Jesse, Jane and Gus (again to varying degrees) reference: the equiprobability (i.e., property of the part) associated with the flip of a fair coin (i.e, the part); the outcome (i.e., the whole) being comprised of five flips of a fair coin (i.e., parts); and, as such, expected, fallaciously, the outcome (i.e., the whole) to have an equal ratio of heads to tails (i.e., the whole to have a property associated with a part). Subsequently, Jesse, Jane and Gus, like Gale, Mike and Saul, utilize their fallacious conclusion, that is, both sequences should have a one to one ratio of heads to tails (as is for the case of the flip of a fair coin), to analyze which of the outcomes they are presented is most "equal" or "even" in terms of their expected ratio. In all three instances above, the outcomes with a four to one ratio of heads to tails is declared too uneven or not equal enough and, thus, is "correctly" determined as least likely to occur.

Concluding Remarks

The fallacy of composition is evident in the response justifications for both normatively incorrect and correct relative likelihood comparisons. For those individuals that incorrectly determined that both events were equally likely to occur, their responses referenced the equiprobability (stated differently by different individuals) of individual coin flips, that the outcomes presented were comprised of five flips of a fair coin and, thus, fallaciously concluded that the events must also be equiprobable. Alternatively, normatively correct relative likelihood comparisons were accompanied with logically fallacious response justifications. The responses from the latter participants also referenced the equiprobability of a single flip of a fair coin, that outcomes were comprised of flips of a fair and, thus, (also) fallaciously concluded that the ratio of heads to tails for the outcomes should as close to one to one as possible. Subsequently, based on this fallacious reasoning, certain participants deemed the outcomes four heads and one tails too "far away" from the expected one to one ratio, whereas other participants deemed the four heads and one tail outcome as too uneven or unequal. As demonstrated, the fallacy of composition is evidenced in both normatively incorrect and correct response justifications.

Discussion

In terms of research investigating relative likelihood comparisons, the use of logical fallacies, particularly the fallacy of composition, is a relatively recent endeavour. However, when the results of the current research are combined with previous research of Chernoff & Russell (2011b), as seen in Table 2, consistencies in the results, numerical and otherwise, are beginning to emerge.

Table 2: Current, Previous And Combined Results

Number of participants	Three heads and two tails	Four heads and one tail	Equally likely
Results of current study			
Class A (26)	1	8	17
Class B (28)	1	6	21
Total (54)	2	14	38
Percentage (100)	**4%**	**26%**	**70%**
Results of previous study			
Total (63)	5	12	46

Percentage (100)	**8%**	**19%**	**73%**
Combined results			
Total (117)	7	26	84
Percentage (100)	**6%**	**22%**	**72%**

Logical fallacies, and, in particular, the fallacy of composition, are proving an effective means when accounting for normatively incorrect, inconsistent and sometime incomprehensible comparisons of relative likelihood (Chernoff, in press, 2012a-c, 2011; Chernoff & Russell, 2012a, 2012b, 2011a, 2011b). As such, and as asserted previously (Chernoff & Russell, 2011b), logical fallacies should be considered, in addition to the traditional theories, frameworks and models, when investigating relative likelihood comparisons (of sequences or events) and teachers' probabilistic knowledge.

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